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Total Rainbow Connected Numbers for Ladder and Steering Amalgamation Graphs J Mucklow

Arbain

Universitas Sembilanbelas November Kolaka, Jl. Pemuda No.336 Kolaka

E-mail: arbaindjingga@gmail.com

Abstract. All graph considered in this paper are finite, simple, and undirected. Let $G = (V(G), E(G))$ be a nontrivial connected graph and k be a natural number. A mapping $c : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ is called a rainbow total-k-coloring if any two vetherertices have distinct colors. The path like that is called a rainbow total-path. Graph G is called rainbow total-connected if any two vertices x and y in $V(G)$ there exist a rainbow total-path $x - y$. The rainbow total-connection number of G, denoted by $trc(G)$, is the smallest number of colors needed to make G rainbow total-connected. Let t be a natural number with $t \geq 2$. Let $\{G_i \mid i \in [1, t]\}$ be a finite collection of graph and each G_i has fixed vertex v_{0i} called a terminal. The amalgamation $Amal(G_i, v_{0i}, t)$ is a graph formed by taking all the G_i 's and identifying their terminals. In this paper is determined lower and upper bounds for the total-rainbow connection number of an amalgamation graph. Additionally, we determine the total-rainbow connection number of amalgamation of ladders and helms.

1. Introduction

Data communication is one very important part of an information society. Especially in the era of computerization as it is today, so much convenience is done by humans in doing their work by communicating data. But behind that convenience, there is a danger that can threaten the personal person, group, and even a country. In it, there are criminal acts carried out by irresponsible people by tapping and accessing data or information from other people, groups or countries that are confidential. Therefore, the security of data communication lines is very important to note.

The collapse of the WTC twin building by the terrorist attacks on 11 September 2001 was one of the many events that occurred due to the absence of secure data communication lines. Based on reports from observations conducted by [1], the attacks of 11 September 2001 could not be anticipated by the government as a result of the failure of coordination between related institutions in the United States. The coordination failure was caused by law enforcement and intelligence agents not communicating data between one and the other. Even if they share data, they only use ordinary data communication lines whose security is not guaranteed. The data that wants to be shared between these institutions are data that are very confidential because they involve national security. In this case, not all institutions are permitted to know or access the entire contents of the data. Therefore, a secure data communication path is needed so that data security is maintained and each institution can only obtain or access data or information from other institutions in accordance with the portion allowed. The path of data communication between these institutions may have to go through other institutions first. Thus, there will be one or more trajectories of information for each of the two institutions. This problem can

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be solved by designing a data communication path, with data communication lines between institutions given a password so that there is no password repetition in that path. This applies to every data communication channel between institutions. Without reducing the effectiveness of the data communication path, many passwords are used. This situation can be modeled in a rainbow connected number introduced by [2].

The graph parameters described by [2]. defined for side coloring in a graph. In addition, [3] define new parameters related to rainbow connected numbers for point coloring on graphs. Another variant of rainbow connectedness, which is total connected rainbows, is defined for side and point staining [4]. Let k be a natural number. The function c $c: V(G) \cup E(G) \rightarrow \{1,2\ldots,k\}$ is said to be rainbow *k*-total colouring in *G*, if for each pair of points $x, y \in V(G)$ there is a path *x* - *y* with each side and each point in the path gets a different colour. Such a track is called the rainbow total trajectory. The graph *G* is called a total connected rainbow if for each pair of points $x, y \in V(G)$ there is a total trajectory of the rainbow *x* - *y*. The rainbow total-connected number denoted by *trc (G)*, is defined as the many minimum colours needed to make a total G-connected rainbow graph. Sun (2013) [5] and Liu et al. (2014) [6] also conducted an investigation regarding the concept of total rainbow connection and obtained the following results.

Lemma 1.1. Let G be a non-trivial connected graph with a order and a fixed diameter (G), then

a. $\mathrm{trc}(G) = 1 \Leftrightarrow G$ is a complete graph;

b. $2\mathrm{diam}(G) - 1 \leq \mathrm{trc}(G) \leq 2n - 3$.

Proposition 1.1. If *G* connecting graph has order n with q point has a degree of at least 2, then $\mathrm{trc}(G) \leq n - 1 + q$. The equation applies if and only if G graphs a tree.

Theorem 1.1. If G graph is not trivially connected with s intersection and t bridge, then $\mathrm{trc}(G) \geq s + t$.

Some of the graphs that have been investigated and obtained by the total connected number of the probe are circle n, graph wheel W_n, and complete bipartite graph $K_{m,n}$

Let $t \in \mathbb{N}$ with $t \geq 2$. $i \in [1, t]$, Let G_i be a non-trivial connected graph, $|V(Gi)| = n_i$, $n_i \in \mathbb{N}$ with $n_i \geq 2$. Suppose $\{G_1, G_2, \ldots, G_t\}$ is a collection of non-trivial connected graphs and each Gi has a fixed point v_{0i}. The Amalgamation Graph G_i is denoted by Amal (G_i, v_{0i}, t) is a graph formed by pasting all the graphs G_i at point v_{0i}. The point v_{0i} is referred to as the terminal point [7].

Arbain (2017) [8] conducted an investigation regarding the total connectivity of rainbows for amalgamation graphs. Some amalgamation graphs that have been investigated and obtained by the cross-linked number are tree amalgamation and complete amalgamation graph.

2. Method

The research method used in this study is as follows.

- Study the literature on the concept of rainbow flowering.
- Constructing several amalgamation graphs.
- Looking for the upper limit of the total connected rainbow number for the amalgamation graph, then look for the total connected number of the rainbow amalgamation graph for the ladder graph class and steering graph.
- Results and Discussion

3. Result and Discussion

3.1 The upper limit of the total connected number of rainbows for amalgamation graph

Theorem 1. Suppose t ∈N, t ≥ 2. Suppose {G_i | i ∈ [1, t]} is a finite collection of non-trivial connected graphs and each G_i has a fixed point v_{0i}. If G≅ Amal (G_i, v_{0i}, t), then 2d (G) -1 ≤ trc (G) ≤1 + $\sum_{i=1}^{t} \mathrm{trc}(G_i)$.

Evidence. First, proved *trc(G) ≥ 2diam(G)-1*. Based on Lemma 1.1, obtained trc (G) ≥2*diam(G)* -1. Next, it shows $trc(G) \leq 1 + \sum_{i=1}^{t} trc(G_i)$

Suppose that f_i' is a trc-coloring (G_i) - total rainbow on G_i.

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Define coloring $f : V(G) \cup E(G) \rightarrow [1,\ 1 + \sum_{i=1}^{t} trc(G_i)]$ with the following rules

- $f(u) = \begin{cases} f_1'(u) & \text{if } u \in V(G_1); \\ f_q'(u) + \sum_{p=1}^{q-1} trc(G_p) & \text{if } u \in V(G_q) \text{ and } q \in [2,t]; \\ 1 + \sum_{p=1}^{t} trc(G_p) & \text{if } u = v_{0i} = v \end{cases}$
- $f(e) = \begin{cases} f_1'(e) & \text{if } e \in E(G_1); \\ f_q'(e) + \sum_{p=1}^{q-1} trc(G_p) & \text{if } e \in E(G_q) \text{ and } q \in [2,t] \end{cases}$

Look at any two different points $x, y \in V(G)$
Case 1 $x, y \in V(G)$ for $x, y \in V(G)$
There is a total rainbow track $x - y$ by coloring *f* that corresponds to f_i'.
Case 2 $x \in V(G_i)$ and $y \in V(G_j)$
For i ∈ [1, t] and j ∈ [1, t] with i = j.
There are total trajectories of rainbows *x* - *v* and *v* - *y* by coloring *f* corresponding respectively to f_i''and f_j', where *v* is the point in *G* corresponding to the terminal point v_{0i}. Let $P_1 = x - v$, $P_2 = v - y$, and $P_1 \cup P_2 = x\text{-}v,\ v - y$, that is, the *x*-*y* path through point *v*. Because the total staining of the rainbow G_i is different for each *i* ∈ [1, *t*], obtained *x*-*v*, *v*-*y* is a rainbow-x total path. Therefore, *f* is the rainbow total coloring on *G*. Then, $trc(G) \leq 1 + \sum_{i=1}^{t} trc(G_i)$

3.2 Numbers connected to total rainbows for staircase amalgamation graphs

The following theorem shows the existence of a lower bound number connected to a total rainbow for an amalgamation graph presented in theorem 2.

Theorem 2. For example $t, n \in \mathbb{N}$ with $t \geq 2$ and $n \geq 3$. For $i \in [1,t]$, for example $G \cong Amal(L_n^i, v_{0i}, t)$ with L_n^i is ladder graph L_n and v_{0i} is point distribution at L_n^i. The number is connected to the total rainbow *G* graph is $trc(G) = 2\ diam\ (G) - 1$.

Evidence. Based on the theorem 1 obtained $trc(G) \geq 2\ diam(G) - 1$

Next, shown $trc(G) \leq 2\ diam(G) - 1$. For example $i \in [1,t]$. Define graph L_n^i with $V(L_n^i) = \{u_{i,j}, v_{i,j} | j \in [1,n]\}$ and $E(L_n^i) = \{u_{i,j}u_{i,j+1}, v_{i,j}v_{i,j+1} | j \in [1, n-1]\} \cap \{u_{i,j}v_{i,j} | j \in [1,n]\}$. Insert $p_i \in [\lceil n/2,\ n \rceil, n]$ without reduce generality, for example v_{i,p_i} is terminal point at L_n^i with $p_1 \geq p_2 \geq p_i$ for every terminal point v_{i,p_i}. For an illustration of a staircase amalgamation graph $Amal(L_n^i, v_{0,i}, t)$ shows in Figure 1. It is easy to check that the diameter *G* is $diam(G) = p_1 + p_2$.

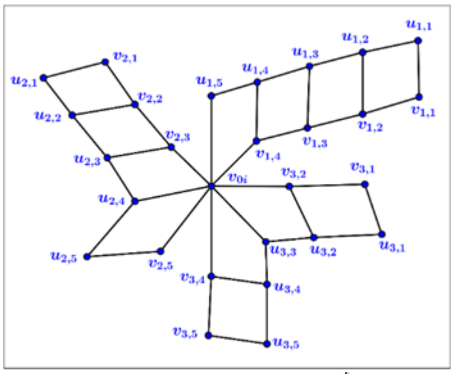

Figure 1. Graph of $Amal(L_5^i, v_{0i}, t)$

Proof is divided into two cases as follows

(i) $p_1 = p_2$

For example $k_i \in [0, n-1-p_i]$ and $l_i \in [0, p_i - 2]$. Define coloring $c : V(G) \cup E(G) \rightarrow [1, 2(p_1 + p_2) - 1]$ with the following rules,

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- $$c(x) = \begin{cases} 2p_1 - 2 - 2k_i & \text{if } x = u_{i,p_i+k_i}; \\ 2p_1 - 2 - 2l_i & \text{if } x = u_{i,p_i-l_i}; \\ 2(p_1 + p_2) - 2 & \text{if } x \in \{u_{i,j}, v_{i,j} \mid j \in \{1,n\} \text{ and } j \neq p_i\}; \\ 2p_1 + 2k_i & \text{if } x = v_{i,p_i+k_i}; \\ 2p_1 + 2l_i & \text{if } x = v_{i,p_i-l_i} \end{cases}$$

- $$c(e) = \begin{cases} 2p_1 - 3 - 2k_i & \text{if } e = u_{i,p_i+k_i} u_{i,p_i+k_i+1}; \\ 2p_1 - 3 - 2l_i & \text{if } e = u_{i,p_i-l_i} u_{i,p_i-l_i-1}; \\ 2p_1 - 1 & \text{if } e = u_{i,p_i} v_{i,p_i}; \\ 2p_1 + 1 + 2k_i & \text{if } e = v_{i,p_i+k_i} v_{i,p_i+k_i+1}; \\ 2p_1 + 1 + 2l_i & \text{if } e = v_{i,p_i-l_i} v_{i,p_i-l_i-1}; \\ 2(p_1 + p_2) - 1 & \text{if } e = u_{i,j} v_{i,j}, j \in [1,n], j \neq p_i. \end{cases}$$

(ii) $p_1 > p_2$
For example $k_i \in [0, n-1-p_i]$, $l_i \in [0, p_i - 2]$, $r \in [0, p_2 - 1]$, and $s \in [1, n - p_2 - 1]$. Define coloring $c : V(G) \cup E(G) \rightarrow [1, 2(p_1 + p_2) - 1]$ with the following rules,

- $$c(x) = \begin{cases} 2p_1 - 2 - 2k_i & \text{if } x = u_{i,p_i+k_i}; \\ 2p_1 - 2 - 2l_i & \text{if } x = u_{i,p_i-l_i}; \\ 2(p_1 + p_2) - 2 & \text{if } x \in \{u_{i,j}, v_{i,j} \mid j \in \{1,n\} \text{ and } j \neq p_i\}; \\ 2p_1 + 2k_i & \text{if } x = v_{i,p_i+k_i}; \\ 2p_1 + 2l_i & \text{if } x = v_{i,p_i-l_i}, i \neq 1; \\ 2p_1 + 2r & \text{if } x = v_{1,p_1-r}; \\ 2s - 1 & \text{if } x = v_{1,p_1-p_2-1-s} \end{cases}$$

- $$c(e) = \begin{cases} 2p_1 - 3 - 2k_i & \text{if } e = u_{i,p_i+k_i} u_{i,p_i+k_i+1}; \\ 2p_1 - 3 - 2l_i & \text{if } e = u_{i,p_i-l_i} u_{i,p_i-l_i-1}; \\ 2p_1 - 1 & \text{if } e = u_{i,p_i} v_{i,p_i}; \\ 2p_1 + 1 + 2k_i & \text{if } e = v_{i,p_i+k_i} v_{i,p_i+k_i+1}; \\ 2p_1 + 1 + 2l_i & \text{if } e = v_{i,p_i-l_i} v_{i,p_i-l_i-1}, i \neq 1; \\ 2p_1 + 2 + 2r & \text{if } e = v_{1,p_1-r} v_{1,p_1-r-1}; \\ 2s & \text{if } e = v_{1,p_1-p_2-1-s} v_{1,p_1-p_2-s}; \\ 2(p_1 + p_2) - 1 & \text{if } e = u_{i,j} v_{i,j}, j \in [1,n], j \neq p. \end{cases}$$

Note the two non-neighboring points $x, y \in V(G)$

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Case 1. $x = u_{h,j}$ and $y = u_{i,k}$
For $h \in [1,t], i \in [1,t], j \in [1,n]$, and $k \in [1,n]$ there is a total trajectory of the rainbow $u_{h,j} - u_{i,k}$, namely $u_{h,j} \dots u_{h,p_h} v_{h,p_h} (v_{i,p_i}) \dots v_{i,k} u_{i,k}$
Case 2. $x = v_{h,j}$ and $y = v_{i,k}$
For $h \in [1,t], i \in [1,t], j \in [1,n]$ and $k \in [1,n]$ there is a total trajectory of the rainbow $v_{h,j} - v_{i,k}$, namely $v_{h,j} u_{h,j} \dots u_{h,p_h} v_{h,p_h} (v_{i,p_i}) \dots v_{i,k}$
Case 3. $x = u_{h,j}$ and $y = u_{h,k}$

- For $h \in [1,t]$ and $j,k \in [1,p_h]$ or $j,k \in [p_h,n]$ with $j < k$ there is a total trajectory of the rainbow $u_{h,j} - u_{h,k}$, namely $u_{h,j} u_{h,j+1} \dots u_{h,k}$
- For $h \in [1,t], j \in [1,p_h]$, and $k \in [p_h,n]$ there is a total trajectory of the rainbow $u_{h,j} - u_{h,k}$, namely $u_{h,j} u_{h,j+1} \dots u_{h,p_h} v_{h,p_h} \dots v_{h,k} u_{h,k}$

Case 4. $x = v_{h,j}$ and $y = v_{h,k}$

- For $h \in [1,t]$ and $j,k \in [1,p_h]$ or $j,k \in [p_h,n]$ with $j < k$ there is a total trajectory of the rainbow $v_{h,j} - v_{h,k}$, namely $v_{h,j} v_{h,j+1} \dots v_{h,k}$
- For $h \in [1,t], j \in [1,p_h]$, and $k \in [p_h,n]$ there is a total trajectory of the rainbow $v_{h,j} - v_{h,k}$, namely $v_{h,j} v_{h,j+1} \dots v_{h,p_h} u_{h,p_h} \dots u_{h,k} v_{h,k}$

For every two points $x, y \in V(G)$ there is a total trajectory of the rainbow $x - y$. Because, $trc(G) \leq 2\,diam(G) - 1$.
As an illustration, coloring is presented $2(p_1 + p_2) - 1$ total rainbow on the ladder amalgamation graph $Amal(L_n^i, v_{0i}, t)$ with $n = 5, t = 3, p_1 = 5, p_2 = 4$ and $p_3 = 3$ as seen at Figure 2.

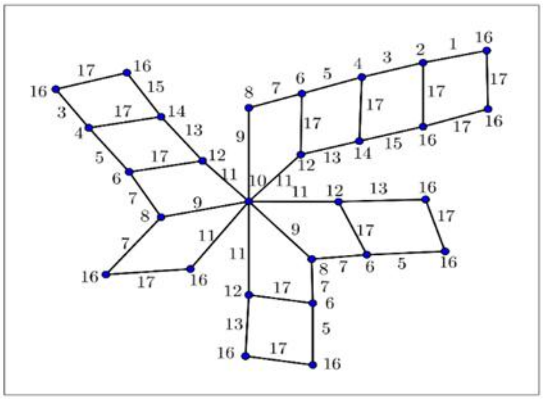

Figure 2. 17 total rainbow coloring of $Amal(L_5^i, v_{0i}, t)$

3.3 The total number of the rainbow is connected to the steering amalgamation graph
The following theorem shows the existence of a boundary over the total connected number of rainbows for an amalgamation graph.
Theorem 3. For example n and t natural number with $n \geq 3$ and $t \geq 2$. For example p, q, and r are integers with $p \in [0,t], q \in [0,t]$, and $r \in [0,t]$ so that $p + q + r = t$. For $i \in [1,t]$, example $G \cong Amal(H_n^i, v_{0i}, t)$ with H_i^n is steering graph of H_n. If the terminal point is the center point, the pendant point, and the points on the H_n arc are respectively as many as p, q, and r, then the total connected number of the rainbow graph G is $trc(G) = 1 + 2n(p + q + r) - r$.
Evidence. Define the set of points and side sets of graph *G* as follows:

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$$
\begin{aligned}
V(G) &= \{v\} \cup \{v_{i,j} | i \in [1,p], j \in [1,n]\} \cup \{w_{i,j} | i \in [1,p], j \in [1,n]\} \cup \\
&\quad \{u_{p+i} | i \in [1,q]\} \cup \{v_{p+i,j} | i \in [1,q], j \in [1,n]\} \cup \\
&\quad \{w_{p+i,j} | i \in [1,t], j \in [1,n-1]\} \cup \{u_{p+q+i} | i \in [1,r]\} \cup \\
&\quad \{v_{p+q+i,j} | i \in [1,r], j \in [1,n-1]\} \cup \{w_{p+q+i,j} | i \in [1,r], j \in [1,n]\} \\
E(G) &= \{v_{i,j}v_{i,j+1}, v_{i,n}v_{i,1} | i \in [1,p], j \in [1,n-1]\} \cup \\
&\quad \{v_{i,j}w_{i,j} | i \in [1,p], j \in [1,n]\} \cup \{vv_{i,j} | i \in [1,p], j \in [1,n]\} \cup \\
&\quad \{v_{p+i,j}v_{p+i,j+1}, v_{p+i,n}v_{p+i,1} | i \in [1,q], j \in [1,n-1]\} \cup \\
&\quad \{v_{p+i,j}w_{p+i,j}, vv_{p+i,n} | i \in [1,q], j \in [1,n-1]\} \cup \\
&\quad \{u_{p+i}v_{p+i,j} | i \in [1,q], j \in [1,n]\} \cup \\
&\quad \{v_{p+q+i,j}v_{p+q+i,j+1}, vv_{p+q+i,1}, vv_{p+q+i,n} | i \in [1,r], j \in [1,n-2]\} \cup \\
&\quad \{v_{p+q+i,j}w_{p+q+i,j}, vw_{p+q+i,n} | i \in [1,r], j \in [1,n-1]\} \cup \\
&\quad \{u_{p+q+i}v_{p+q+i,j}, vu_{p+q+i} | i \in [1,r], j \in [1,n-1]\}.
\end{aligned}
$$

As an illustration, it is presented a steering amalgamation graph $Amal\,(H_4^i, v_{0i}, t)$ in Figure 3, with $i \in [1,4]$ terminal point H_4^1 is the center point, terminal point H_4^2 and H_4^3 is a pendant point , and the terminal point H_4^4 is the point in the arc H_4.

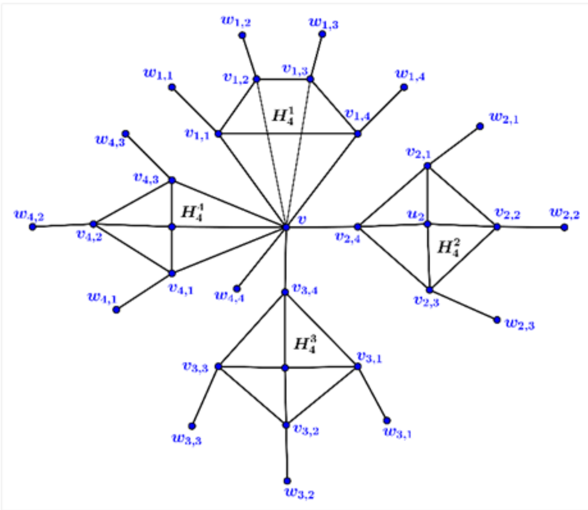

Figure 3. Graph of $Amal(H_4^i, v_{0i}, t), i \in [1,4]$

First, indicated, $trc(G) \geq 1 + 2n(p+q+r) - r$

Note that graph G has a cutoff of $1 + n(p+q+r) - r$ and a bridge of $n(p+q+r)$. Based on theorem 1.1 obtained $trc(G) \geq 1 + 2n(p+q+r) - r$

Next is shown $trc(G) \leq 1 + 2n(p+q+r) - r$

Define the coloring $c : V(G) \cup E(G) \to [1, 1 + 2n(p+q+r) - r]$ with the following rules

- $c(x) = \begin{cases} 2n(i-1) + 2j & \text{if } x = v_{i,j}, i \in [1, p+q] \text{ and } j \in [1.n]; \\ 2n(p+q+i-1) + 2j - i + 1 & \text{if } x = v_{i,j}, i \in [1,r] \text{ dan } j \in [1, n-1]; \\ 2n(i-1) + 2 & \text{if } x = u_i, i \in [p+1, p+q]; \\ 2n(p+q+i-1) + 3 - i & \text{if } x = u_i, i \in [1,r]; \\ 1 + 2n(p+q+r) - r & \text{if } x = v. \end{cases}$

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- $c(e) = \begin{cases} 2n(i-1)+2j-1 & \text{if } e = v_{i,j}w_{i,j}, i \in [1,p+q] \text{ and } j \in [1,n] \text{ with } v_{i,n} = v; \\ 2n(i-1)+2j-3 & \text{if } e = u_i v_{i,j}, i \in [1,p+q] \text{ and } j \in [2,n] \text{ with } u_i = v \text{ for } i \in [1,p]; \\ 2ni-1 & \text{If } e = u_i v_{i,1}, i \in [1,p+q] \text{ with } u_i = v \text{ for } i \in [1,p]; \\ 2n(p+q+i)-1 & \text{if } e = u_i v_{i,1}, i \in [1,r]; \\ 2ni+1-j & \text{if } e = v_{i,j}v_{i,j+1}, i \in [1,p+q] \text{ and } j \in [1,\lceil n/2 \rceil]; \\ 2n(p+q+i)+1-i-j & \text{if } e = v_{i,j}v_{i,j+1}, i \in [1,r] \text{ and } j \in [1,\lceil n/2 \rceil]; \\ 2n(i-1)+3\lceil n/2 \rceil+1-j & \text{if } e = v_{i,j}v_{(i,j+1)}, i \in [1,p+q] \text{ and } j \in \left[\left\lceil \frac{n}{2} \right\rceil + 1, n\right] \end{cases}$
- $c(u_i v_{i,j}) = 2n(p+q+i-1)+2j-i-2$ for $i \in [1,r]$ and $j \in [2,n]$;
- $c(v_{i,j}v_{i,j+1}) = 2n(p+q+1-1)+3\left\lceil \frac{n}{2} \right\rceil + 2 - j - i$ for $i \in [1,r]$ and $j \in [\left\lceil \frac{n}{2} \right\rceil + 1, n]$ with $v_{i,n} = v$

As an illustration, a total of $1+2n(p+q+r)-r$ total rainbow is presented on the steering amalgamation graph $Amal(H_n^i, v_{0i}, t)$ with $i \in [1,4], n = 4, p = 1, q = 2$ and $r = 1$ in figure 4. It will be shown for every two points $x, y \in V(G)$ there is a total path of rainbow $x - y$. Obviously for any two neighboring points $x, y \in V(G)$ there is a total trajectory of the rainbow *x-y*, i.e. *xy*. Notice the two different points not neighboring $x, y \in V(G)$.

Case 1. $x = w_{h,j}$ and $y = w_{i,k}$

- For $h \in [1,p], i \in [1,p], j \in [1,n]$, dan $k \in [1,n]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}vv_{i,k}w_{i,k}$.
- For $h \in [1,p], i \in [1,q], j \in [1,n]$, dan $k \in [2,n-2]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}vv_{i,n}u_i v_{i,k}w_{i,k}$.
- For $h \in [1,p], i \in [1,q], j \in [1,n]$, dan $k \in [1,n-1]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}vv_{i,n}v_{i,k}w_{i,k}$.
- For $h \in [1,p], i \in [1,r], j \in [1,n]$, dan $k \in [2,n-2]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}v\,u_i v_{i,k}w_{i,k}$.
- For $h \in [1,p], i \in [1,r], j \in [1,n]$, dan $k \in [1,n-1]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}vv_{i,k}w_{i,k}$.
- For $h, i \in [1,q]$ dan $j, k \in [2,n-2]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}u_h v_{h,n}vv_{i,n}u_i v_{i,k}w_{i,k}$.
- For $h, i \in [1,q]$ dan $j, k \in [1,n-1]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}v_{h,n}vv_{i,n}v_{i,k}w_{i,k}$.
- For $h, i \in [1,q]$, $j \in [2,n-2]$, dan $k \in [1,n-1]$ there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}u_h v_{h,n}vv_{i,n}v_{i,k}w_{i,k}$.
- For $h \in [1,q], i \in [1,r]$, dan $j, k \in [2,n-2]$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}u_h v_{h,n}vu_i v_{i,k}w_{i,k}$.
- For $h \in [1,q], i \in [1,r]$, dan $j, k \in \{1,n-1\}$, there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}v_{h,n}vv_{i,k}w_{i,k}$.

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- For $h \in [1, q], i \in [1, r]$, $j \in [2, n-2]$, dan $k \in \{1, n-1\}$ there is a total trajectory of the rainbow $w_{h,j} - w_{i,k}$, namely $w_{h,j}v_{h,j}u_h v_{h,n} v v_{i,k} w_{i,k}$.

Case 2. $x = w_{i,j}$ and $y = w_{i,k}$

- For $i \in [1, t]$ dan $j, k \in [1, n]$ dengan $|j - k| = 1$, there is a total trajectory of the rainbow $w_{i,j} - w_{i,k}$, namely $w_{i,j}v_{i,j}v_{i,k}w_{i,k}$.
- For $i \in [1, t]$ dan $j, k \in [1, n]$ dengan $|j - k| \geq 2$ dan $j, k \neq 1$, there is a total trajectory of the rainbow $w_{i,j} - w_{i,k}$, namely $w_{i,j}v_{i,j}u_i v_{i,k} w_{i,k}$.
- For $i \in [1, t]$, $j = 1$, dan $k \in [3, \frac{n}{2} + 1]$, there is a total trajectory of the rainbow $w_{i,j} - w_{i,k}$, namely $w_{i,j}v_{i,j}v_{i,j+1} \dots v_{i,k}w_{i,k}$.
- For $i \in [1, t]$, $j = 1$, dan $k \in [\frac{n}{2} + 2, n]$, there is a total trajectory of the rainbow $w_{i,j} - w_{i,k}$, namely $w_{i,j}v_{i,j}v_{i,n}v_{i,n-1} \dots v_{i,k}w_{i,k}$.

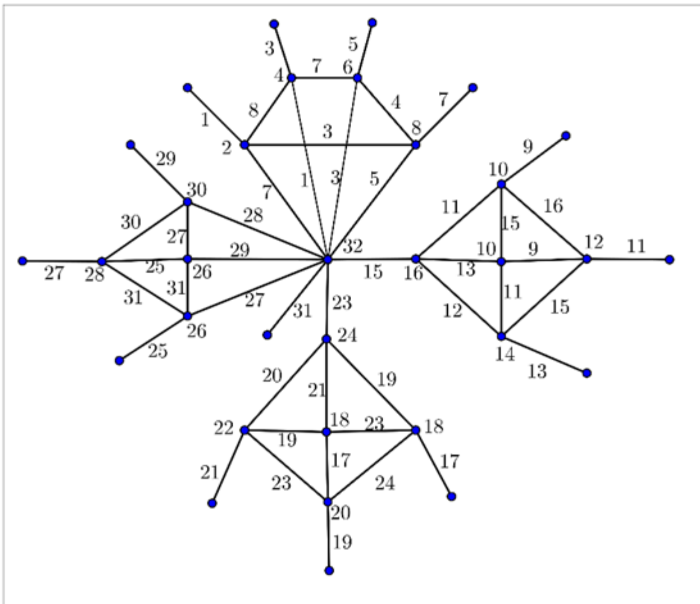

Figure 4. 32 total rainbow coloring on $Amal(H_4^i, v_{0i}, t)$

4. Conclusion

Suppose that t and n are natural numbers with $t \geq 2$ and $n \geq 3$. For $i \in [1, t]$, suppose $G \cong Amal(G_i, v_{0i}, t)$ with G_i is a non-trivial connected graph. Suppose the diameter G is $diam(G)$. The conclusions that can be drawn are as follows:

- The total connected number of the rainbow for graph G satisfies $2diam(G) - 1 \leq trc(G) \leq 1 + \sum_{i=1}^{t} trc(G_i)$
- For example, G_i is a graph of L_n ladder with number n. The total connected number of the rainbow graph G is $trc(G) = 2diam(G) - 1$.
- Suppose that G_i is the order of the H_n steering graph n. The total connected number of the rainbow graph G is $trc(G) = 1 + 2n(p + q + r) - r$ with p, q and r successively declaring that many H_n have a terminal point in the form of a center point, a pendant point, and a point on arc H_n

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