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HEURISTIC STRATEGIES AS COGNITIVE BRIDGES IN SOLVING ARITHMETIC SEQUENCE WORD PROBLEMS: A CASE STUDY

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ABSTRACT

This study explores the micro-process dynamics of students in applying heuristic strategies to arithmetic sequence word problems through a qualitative case study. Using Think Aloud with the subject YD, the results show a paradox between procedural fluency with formulas and cognitive failure when switching to manual strategies. This transition triggers cognitive overload and representational misconceptions, where the subject is distracted by visuals, resulting in inconsistent answers. This study concludes that without a solid conceptual foundation, heuristic instruction risks weakening students' confidence and triggering logical errors, leading to a loss of focus in their work.

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1. INTRODUCTION

Modern mathematics education emphasizes problem-solving skills as a core competency, requiring students not merely to perform rapid calculations or memorize procedures, but to think strategically and creatively when facing non-routine situations. (Szabo et al., 2020). This competency requires students to think strategically, creatively, and to be able to deal with new situations that cannot be solved only with routine algorithms (Xu et al., 2023). However, real-world data show that word problems, as the primary instrument for evaluating problem-solving, often pose the greatest cognitive obstacle for students

(Supriadi et al., 2024). Many students who are proficient at calculating still struggle with interpreting information and devising strategies for solving word problems (Kolar & Hodnik, 2021). This failure is often not caused by an inability to calculate, but rather by weaknesses in interpreting information, understanding text, scientific knowledge, and transferring from verbal language to mathematical models (Kaitera & Harmoinen, 2022). Therefore, students' mastery of mathematics cannot be measured solely by the final result, but rather by how their thinking process is formed when facing mathematical challenges. One mathematical topic that strongly embodies these challenges in problem-solving is arithmetic sequences, particularly when presented in the form of word problems.

At the secondary school level, arithmetic sequences are one of the topics that are rich in applications relevant to everyday life, such as calculating savings interest, production growth, or patterns of object arrangement (Wakhata et al., 2022). This material has unique characteristics because it requires students to identify patterns of regularity from a set of data presented implicitly in a narrative. Complexity arises when students get stuck using formal formulas without understanding the underlying structure of the problem (Kutaka et al., 2024). As a result, when the questions are slightly altered from the routine examples, students lose their way because they rely solely on memorizing formulas without a flexible understanding of the concepts.

To overcome this complexity, an approach that goes beyond standard algorithmic steps is needed, namely a heuristic strategy. Heuristic strategies refer to non-algorithmic approaches that guide learners through understanding the problem, planning, executing, and reflecting on solutions, as emphasized in Polya's classical problem-solving framework (Torres-peña et al., 2025). In the context of arithmetic sequences, heuristics can take the form of strategies such as sketching, guessing and testing, simplifying problems, or working backwards (Amrullah et al., 2024). This approach encourages exploration, discussion, and reflection, so that students do not just rely on formulas but also understand the structure and context of the problem. George Polya, in his problem-solving theory, emphasizes the importance of heuristic steps such as understanding the problem, planning, executing, and looking back (Gopinath & Lertlit, 2022). This strategy serves as a cognitive bridge that helps students simplify the complexity of word problems into a more manageable form.

Previous studies have shown the positive impact of applying heuristic strategies. Heuristic-based worked examples have been shown to improve strategic knowledge and mathematical modeling competence, especially in upper middle school students (Raihani et al., 2025). The active application of heuristic strategies significantly improves students' attitudes toward mathematics, enhances collaboration, communication, and conceptual understanding (Zamnah et al., 2021). The majority of studies on students' mathematical problem-solving abilities are still dominated by quantitative approaches that assess general learning outcomes or the effectiveness of specific learning models (Muhaimin et al., 2024).

Qualitative studies that investigate problem-solving processes tend to focus on error analysis or comparisons across ability groups, rather than examining the micro-level dynamics of individual heuristic strategy use (Herold-Blasius, 2024). Research that specifically explores thinking processes and heuristic strategies at the individual level, especially in the context of arithmetic sequence story problems, are still very rare, and there is also very limited research that specifically highlights the micro-processes or dynamics of the use of heuristic strategies in individuals in depth (case studies), particularly in exploring how a student builds mental strategies when solving arithmetic sequence story problems (Khalid et al., 2020). Although recent studies have begun to explore individual heuristic profiles using process-oriented data such as audiovisual recordings (Favier & Dorier, 2024), empirical evidence detailing how a single student constructs, shifts, and adapts heuristic strategies during problem solving is still limited. Despite extensive research on mathematical problem-solving, most studies remain dominated by quantitative approaches that evaluate learning outcomes or instructional effectiveness. Qualitative investigations tend to focus on error analysis or group comparisons, leaving the micro-process dynamics of individual heuristic strategy construction largely unexplored, particularly in arithmetic sequence word problems.

This study focuses on a single student named YD from a senior high school in East Java. The selection of the subject was based on unique characteristics in problem solving, where YD showed a tendency to use intuitive and varied heuristic strategies, such as relying on visual representations before entering formal formulas. The uniqueness of this cognitive profile offers novelty in understanding how the complexity of arithmetic sequence story problems is unraveled through an authentic heuristic approach. This research also offers novelty by positioning heuristic strategies not merely as problem-solving tools, but as vital cognitive bridges that connect verbal narrative comprehension with abstract algebraic formalism. To uncover this mechanism, this study was specifically designed to answer the research question of how subjects construct initial heuristic strategies to identify implicit patterns in story problems, how the process of representation transformation occurs when subjects change from a visual-intuitive approach to a formal mathematical model, and how these strategies are reused as metacognitive verification tools when subjects encounter dead ends. Thus, this exploration not only describes the steps to the solution but also dissects the micro-process dynamics of the transition from informal to procedural thinking, which is often overlooked in mathematics learning.

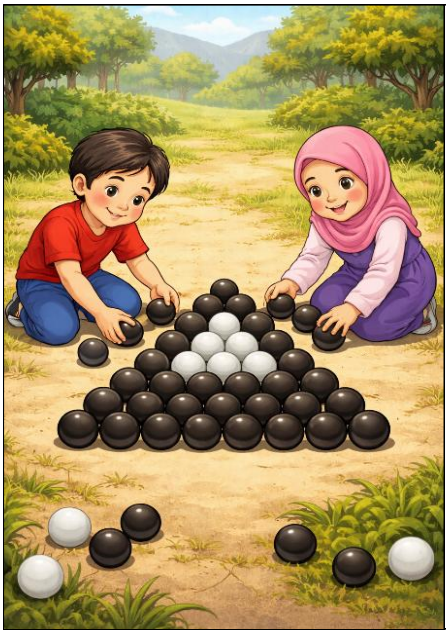
Based on this background, this study aims to explore and describe in depth the micro-process dynamics of subjects' cognition, with a primary focus on revealing the mechanism of heuristic strategies as a cognitive bridge that facilitates the crucial transformation from visual-intuitive representations to formal mathematical models in solving arithmetic sequence story problems. Through an in-depth analysis of this transition, the study makes a significant theoretical contribution by enriching the literature on representation translation and unique thinking processes, confirming the widely accepted view that the construction of informal understanding is a fundamental prerequisite that must be established before procedural algorithmic mastery can operate effectively. Simultaneously, this study offers strategic practical implications for the development of mathematics pedagogy, namely, providing a basis for educators to design more adaptive learning interventions that accommodate students' diverse heuristics as valid stages for building problem-solving maturity, rather than simply imposing formula standardization from an early age.

2. METHOD

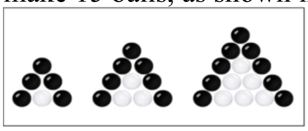
This study was conducted using a qualitative approach with a case study design, which was chosen because the urgency of the research focused on an in-depth exploration of cognitive processes and mental strategies that cannot be measured solely through statistical figures. The main focus of the study was directed at analyzing the micro-processes of problem solving to reveal how heuristic strategies are constructed in dealing with the complexity of arithmetic sequence story problems. The subject in this study was determined using purposive sampling, namely a student named YD from a senior high school in East Java. The selection of YD as the sole subject was methodologically validated as an information-rich case, in which the subject had unique and specific characteristics that were essential to the research objectives. YD was chosen because she met the criteria as a critical case with excellent verbal articulation skills and a strong tendency to use non-algorithmic and intuitive strategies. These idiosyncratic characteristics allowed the researcher to obtain depth of data that would have been impossible to achieve through a broad sample, enabling the phenomenon of strategy transition to be captured in its entirety.

Data collection was conducted interactively and naturalistically by placing the researcher as a key instrument supported by test and interview guidelines. The main techniques applied were the integration of problem-solving tests and the Think Aloud method. To minimize bias and ensure that the verbal data obtained purely reflected the thinking process, prior to core data collection, the subject was given a Think Aloud simulation session using simple practice questions. This step aims to accustom subject to voicing everything they think, such as doubts, strategy revisions, or mental visualizations, naturally, without additional cognitive burden when working on actual non-routine arithmetic sequence story problems. Immediately after the test, task-based interviews were conducted to confirm the researcher's interpretation of the steps taken. Data validity was ensured through triangulation techniques by comparing the consistency between written answers, verbal recordings, and interview results, as well as with the help of audio-visual recording devices to capture the subjects' gestures and expressions.

Data analysis was conducted inductively and continuously following the interactive model of Miles, Huberman, and Saldana, which includes data condensation, data presentation, and conclusion drawing. The process began with transcribing all verbal data from Think Aloud and interviews, then selecting relevant information. Data coding was carried out strictly with reference to the operational indicators of heuristic strategies adapted from Polya's stages, namely: (1) identification of information to understand the problem, (2) formulation of strategies (sketches/patterns) in planning the solution, (3) execution of procedures in implementing the plan, and (4) evaluation of results in the review stage. The data were then presented in a matrix that juxtaposed written answers and verbal thinking processes to systematically map YD's cognitive structure. The final stage of the analysis culminated in identifying the dominant heuristic patterns used by the subject in unraveling the complexity of the problem. The question instruments used in this study were adopted based on Purwasih (2024), a reference on PISA question writing, where the question details and data analysis flow are presented in the figure below.



Two siblings from East Java play with clay balls arranged in a triangle. The clay balls are made from a mixture of clay and recycled waste. The balls are painted black and white. In the first sequence, Zahra needs five black clay balls and one white clay ball to form a triangle. Then, in the second sequence, she needs seven black balls and three white balls. In the third sequence, she needs nine black balls and six white balls. To fill the available space, Zahra will make 15 balls, as shown in the picture below.



Is there another way to determine the number of black circles in the 40th layout besides continuing to draw? If so, explain and elaborate on your answer!

Figure 1. PISA Questions

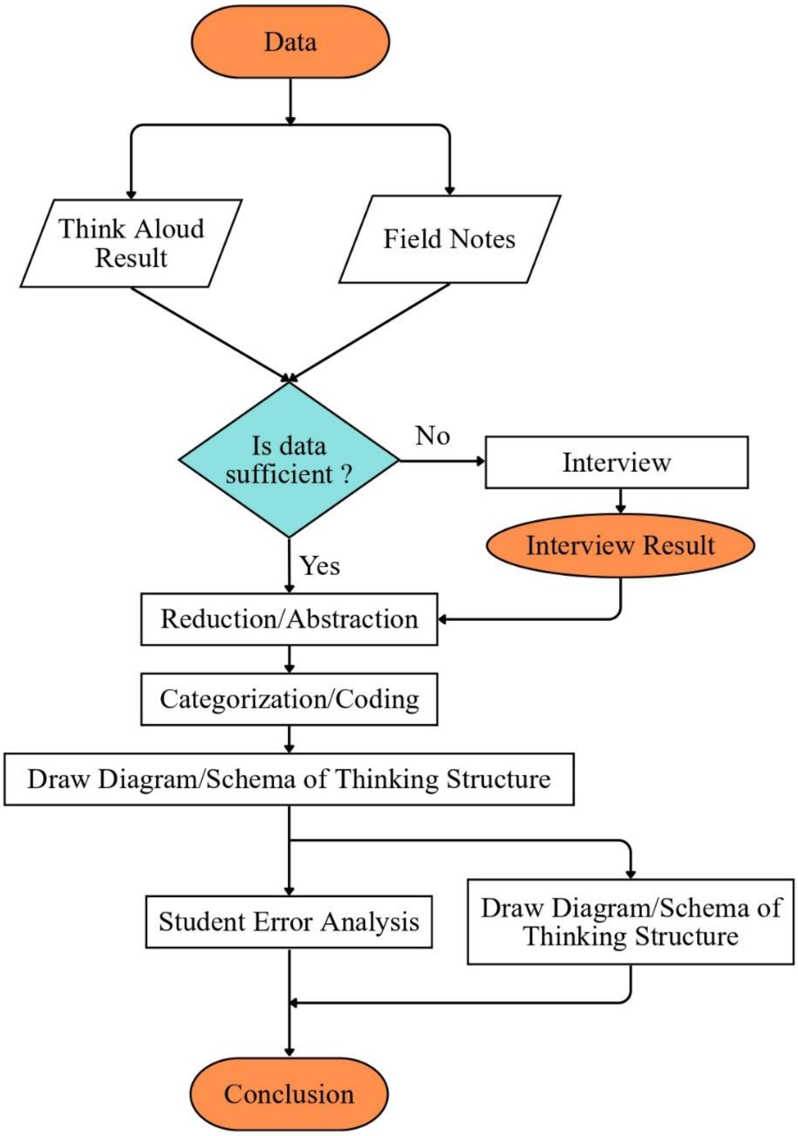


Figure 2. Data Analysis Flowchart

3. RESULTS AND DISCUSSION

3.1. Results

Based on the results, there was a significant difference in problem-solving characteristics between the application of algorithmic strategies and manual heuristic strategies. In the initial stage, YD was able to demonstrate a structured solution flow using formal formulas, which produced the correct answers. Subsequently, when the participant shifted to a heuristic strategy in the form of a manual listing in response to the task instruction, the verbal data showed a change in the expression of solution steps, while the written work displayed a less organized presentation of numerical elements. This shift in strategy was consistently captured across both verbal data and written artifacts. The results of YD's task completion, which recorded the traces of this strategy transformation, are presented in the figure below.

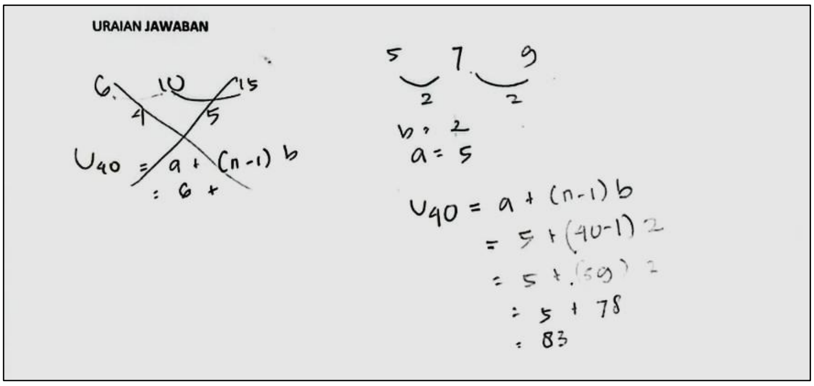


Figure 3. YD's Work Results (Written Solution Using Formula-Based Strategy)

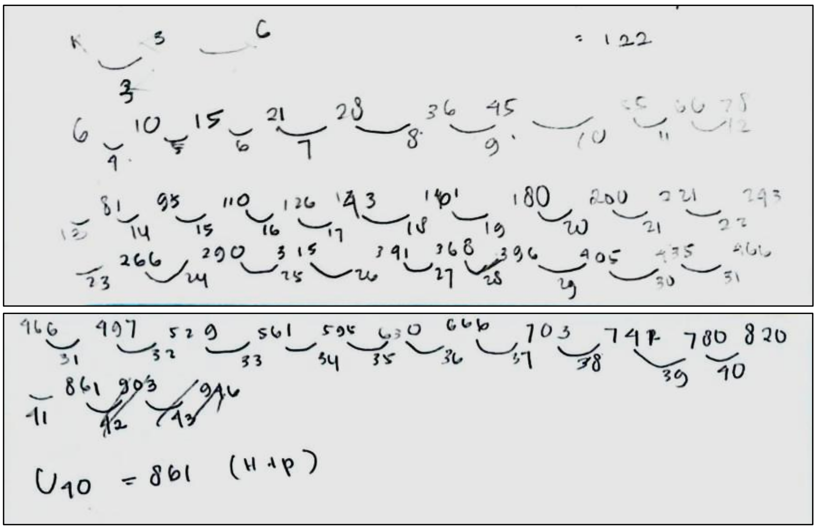


Figure 4. YD's Work Results (Written Work Using Manual Listing Strategy)

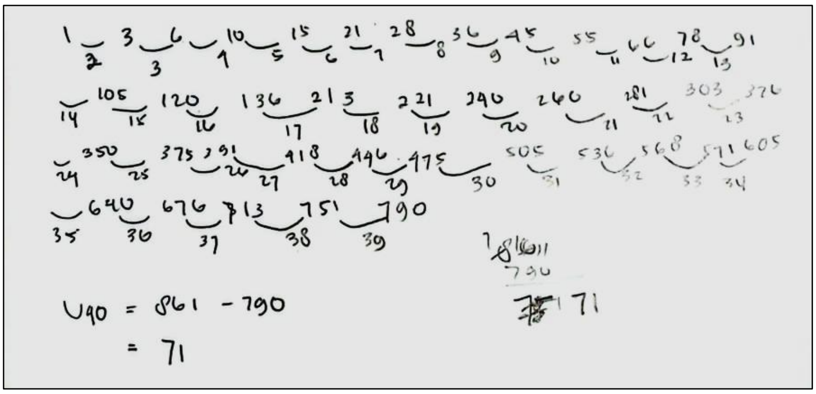


Figure 5. YD's Work Results (Written Work Using Manual Listing Strategy)

In addition to written evidence, this study also recorded the dynamics of the subjects' thinking processes through the Think Aloud method to reveal invisible mental activities. This study provides results by comparing verbal transcripts, physical writing activities, and interpretations of cognitive behavior to map the stages of the subjects' problem solving, from successful initial identification to the emergence of final conflict. The findings show a shift in the subject's psychological condition from confidence to anxiety as the manual strategy applied failed. Details of the results of the think-aloud method, behavior, and cognition are described in the following table.

Table 1. Think Aloud Method Results

Stage	Think Aloud Transcript (reconstruction)	Writing Activity (Documentary Evidence)	Behavior & Cognition
Initial Identification (Successful)	The student works confidently and enthusiastically, muttering to himself, "Okay, the question is about the black ball. The pattern is 5, 7, 9... That means the difference is 2. Just use the Un formula to make it faster."	Writing the formula Un = a + (n-1).b and calculating 5+(39)2=83 in Figure 3.	(Confident) The subject is confident with the routine procedure. The subject completed the task by directly substituting values into the formula.
Heuristic Exploration (Beginning to Doubt)	The student begins to rethink the question and says "The question asks for another method... try sorting them one by one. Or calculate the total number of balls first? 6, 10, 15..."	They begin scribbling down the total number of balls (6, 10, 15, 21...) in Figure 4.	(Confusion) The subjects lose focus on the "black balls" and switch to counting the "total number of balls" (black + white). A misconception occurs in the new strategy.
Implementation of Manual Strategy (Crisis)	Students began to feel confused and scratch their heads, then muttered "Oh no, why are the numbers getting bigger? Keep going until 40. ..., 36, 45... oh my, there are so many calculations."	They write a long series of numbers until the page is full. The writing becomes slanted and messy.	(Cognitive Overload) The subject wrote an increasing sequence of numbers without apparent organization. Subjects become exhausted from manual calculations, increasing the potential for human error.
Final Results & Conflict	The students began to fidget, looking left and right while saying "The answer is 71. Huh? Why is the result different from 83? Is the formula wrong, or did I make a mistake in my manual calculation? How can it be 71?"	They wrote $U_{40} = 71$ hesitantly, as shown in Figure 5.	(Anxiety Triggered) The disparity between the results 83 (Correct) and 71 (Incorrect) triggered panic. The subjects were unable to verify which was correct.

To validate the findings from observations and written tests, task-based interviews were conducted to explore the reasons behind the inconsistencies in the subjects' answers. The interview data revealed that the instruction to find another way triggered self-doubt, causing the subjects to abandon the correct formula procedure and switch to a manual method that was prone to error. The subjects also confirmed that they were confused in distinguishing between the pattern of black balls and the total number of balls when performing manual calculations. The interview excerpts describing the subjects' acknowledgments are summarized in the table 2.

Table 2. Task-Based Interview Results

No	Interview Questions	Answer Results
1	YD, I see that on page 1 you wrote the answer 83. But on page 3, you scribbled a lot of numbers until you got 861, and the final result was 71. Can you tell me what happened?	Yes, sir. At first, I was sure that using the formula, the result was 83. I found it quickly. But the question asked, “Is there another way?” So I tried sorting the balls manually one by one. I thought that if I calculated it manually, the result would be the same. It turned out to be different, and I was confused.
2	Then, why did the numbers reach hundreds?	Well, sir, when I was doing the manual calculation, I got confused about which balls were black and which were white. I ended up adding them all up (the total number of balls). The further down I went, the bigger the numbers got, and I started to panic. My hand got tired from writing. I thought, ‘Why does the manual method turn out so strange?’ I became afraid that my formula answer of 83, which used a simple method, might be wrong. It's different from 71, which used a long method that seems correct.
3	How did you feel when you saw that your manual result (71) was different from the formula result (83)?	I was nervous, sir. I was confused about which one to choose. I thought that because the manual method required more effort (it took longer), it must be correct. I became anxious and lost confidence in the formula I had used earlier. My mind went blank.

3.2. Discussion

The results of the analysis of the subjects' problem-solving activities show a dominance of procedural abilities in contrast to the fragility of heuristic strategies. In the initial identification stage, subjects were able to solve problems quickly and accurately using the formal formula for arithmetic sequences U_n. This speed of execution indicates that subjects had reached the automation stage in routine algorithms, in line with Nadilia & Wijayanti's (2023) view of mastery of basic procedures. However, this success is pseudo-conceptual because once the subject is asked to use an alternative (heuristic) strategy, they lose their conceptual grasp. This confirms that fluency in calculation does not always guarantee a deep understanding of the problem structure necessary for flexible thinking.

The transition from an algorithmic approach to a heuristic approach became a critical point of failure for the subject. Think Aloud data recorded significant verbal hesitation when the subject tried to break away from formal formulas. Instead of using heuristics to find simple patterns (e.g., looking at the difference between terms), the subject was stuck in an inefficient manual listing strategy. This phenomenon reflects the cognitive barriers described by Basir & Aminudin (2020), where students have difficulty transferring from algorithmic understanding to independent strategy modeling. In YD's case, the heuristic strategy did not function as an exploration aid but rather became an additional burden due to the absence of a structured thinking scheme.

A more in-depth analysis of the answer sheet shows a fatal error in problem representation when the subject applied her heuristic strategy. The subject experienced a misconception by shifting her focus from finding the n-th term of the black ball pattern to finding the total number of balls (arithmetic sequence), as seen from the sequence of numbers 6, 10, 15... that she wrote. This error shows that the visual-intuitive strategy developed by the subjects did not align with the logic of the problem, but instead created a misleading divergent line of thinking. This finding reinforces the argument of Taufik et al. (2022) that the biggest challenge in story problems is not the calculation but the translation of verbal narratives into accurate mental models.

The application of a brute force manual strategy (listing one by one up to the 40th term) causes cognitive overload. This differs from the theory that heuristics serve to simplify problems Indah et al. (2021). However, the findings in this case show the opposite, indicating that immature heuristic strategies actually complicate the solution process. The subjects' increasingly messy handwriting and verbal complaints about the large number of calculations were physical indicators of mental fatigue. Without the ability to generalize patterns, the subjects' working memory was drained to process massive amounts of numerical data, blurring their focus on the main goal of problem-solving.

The affective impact of the failure of the heuristic strategy was clearly seen in the emergence of math anxiety when the subject found a difference in the final results. The conflict between the formula result (83) and the manual calculation result (71) triggered verbal panic, in which the subject questioned the validity of both answers without the ability to verify them logically. This condition is in line with the findings of Kaitera & Harmoinen (2022) that affective factors greatly influence problem-solving performance. However, in the context of this case study, the anxiety was specifically triggered by the subjects' unpreparedness in managing the cognitive dissonance that arose from the use of two conflicting strategies.

Metacognitive weaknesses were the main focus in the review stage, which should have been the final step in Polya's heuristic strategy. The subject did not recheck the logic of his calculations but only doubted the final result. This failure of control function prevented the subject from detecting that the manual strategy she used (adding up the totals) actually answered a different question than the formula she used. This indicates that the use of heuristic strategies in students with characteristics such as YD's has not yet reached a mature metacognitive stage, but is still at the trial-and-error stage, which is prone to errors.

In dialogue with previous literature, these findings provide a contextual nuance to claims about the benefits of heuristics. While the research by Saiful et al. (2020) emphasizes that heuristics encourage deep exploration, this case study offers the limitation that such exploration must be based on a correct understanding of the pattern concept. Without a basic understanding of arithmetic sequences, the freedom to use heuristic strategies can actually lead students into tedious and ineffective manual procedures. Therefore, heuristic strategies cannot be viewed as stand-alone strategies, but rather require strong integration with conceptual understanding.

The results of an in-depth interview with YD revealed that the inconsistency in her answers was not caused by an inability to perform basic calculations, but rather by psychological doubts about the validity of her procedural answers. YD explicitly acknowledged that the instruction to find another way was the main trigger that shook her confidence in the initial algorithmic solution (83), causing her to feel pressured to validate it through manual methods, which she considered more authentic. When asked about the origin of the hundreds digit, YD revealed that her visual focus was distracted by the shape of the triangle as a whole, causing her to unconsciously calculate the total number of balls by following the pattern of the triangle numbers rather than sorting only the black balls. This admission validates the finding that the transition from symbolic representation to visual representation without a strong conceptual foundation actually traps students in representational misconceptions, where they fail to distinguish between relevant information and visual distractions.

Furthermore, YD's narrative highlights the significant emotional impact of the excessive cognitive load that arises when this manual strategy is employed. She describes the moment when she realized that her calculations had reached hundreds as a situation that triggered panic and intense cognitive dissonance, given the extreme disparity in numbers compared to the results of her previous formula calculations. Although she intuitively realized that the answer 83 felt more logical, the mental exhaustion from the process of adding up the series of numbers to the 40th term had paralyzed her critical ability to re-verify. In the end, YD tended to resign herself to accepting the incorrect manual calculation simply because of the amount of energy she had expended, a phenomenon that confirms how focus saturation can effectively alter students' metacognitive control functions in complex problem-solving situations.

As a conclusion from this micro-process analysis, YD's problem-solving profile represents a student who is procedurally competent but not yet strategically adaptive. This finding implies that learning interventions are not sufficient with just routine practice questions, but need to emphasize explicit heuristic strategy training, such as creating pattern tables or simple visualizations, as cognitive bridges before entering formal formulas. Thus, the generalization of these findings is limited to similar subject characteristics, but it provides important implications for educators to be more aware of the illusion of student competence who are only proficient in using formulas but fragile when faced with variations in strategies.

4. CONCLUSION

This study highlights an important gap between procedural fluency and conceptual understanding in solving arithmetic sequence word problems. Although routine algorithms yielded correct answers, the use of manual heuristics exposed instability in the subject’s thinking structure, leading to increased cognitive load and conceptual confusion between determining the nth term and calculating the sum of the sequence. These findings indicate that heuristic strategies, when not grounded in well-internalized pattern concepts, may undermine problem-solving effectiveness and students’ confidence rather than support meaningful understanding. Consequently, heuristic mastery should be positioned as a foundational cognitive bridge that fosters strategic adaptability and deep reasoning, rather than as a secondary or alternative technique appended to procedural instruction. Therefore, students must have reasoning skills that are defined as a conscious effort to understand the structure of a problem in depth, so that they have robust logical verification tools and do not simply rely on procedural memorization, which is prone to collapse when faced with cognitive distortions or unexpected variations in questions. This study is limited to a single case, and therefore, its findings cannot be generalized to all students but are transferable to learners with similar cognitive characteristics.

Based on these findings, mathematics instruction should prioritize the systematic development of heuristic strategies as a foundational component of conceptual understanding rather than as a supplementary alternative to procedural algorithms. Teachers are encouraged to design learning experiences that emphasize pattern recognition, meaningful representations, and reflective verification processes to strengthen students' strategic adaptability and reasoning coherence. Future studies are recommended to involve multiple subjects with diverse cognitive profiles to further explore the development and maturation of heuristic strategies in different mathematical contexts.

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